

# Speed of field driven domain walls in nanowires with large transverse magnetic anisotropy

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## Abstract

Recent analytical and numerical work on field driven domain wall propagation in nanowires has shown that for large transverse anisotropy and sufficiently large applied fields the Walker profile becomes unstable before the breakdown field, giving way to a slower stationary domain wall. We perform an asymptotic expansion of the Landau Lifshitz Gilbert equation for large transverse magnetic anisotropy and show that the asymptotic dynamics reproduces this behavior. At low applied field the speed increases linearly with the field and the profile is the classic Landau profile. Beyond a critical value of the applied field the domain wall slows down. The appearance of a slower domain wall profile in the asymptotic dynamics is due to a transition from a pushed to a pulled front of a reaction diffusion equation.

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Magnetic domain wall propagation is an active area of research both as an interesting physical phenomenon as well as for its possible applications in logic devices, magnetic memory elements and others [1]. The dynamics of magnetic domain walls is described by the Landau Lifshitz Gilbert (LLG) equation [2, 3] which cannot be solved analytically except in very special cases. For an infinite medium with uniaxial anisotropy and an external field applied along the symmetry axis, the Walker solution [4] provides the best known analytical expression for the profile and speed of the domain wall. The exact Walker solution, predicts that the speed increases linearly with the field up to a critical field  $H_w$ . Above this value a sudden drop in velocity and an irregular precessing motion of the magnetization appears. Field induced domain wall propagation in thin films and nanowires has been examined with greater detail in recent work. The numerical study [5] showed that depending on the relative magnitude of hard axis anisotropy different scenarios arise. For small hard axis anisotropy the Walker solution is realized. For sufficiently large values of the hard axis anisotropy the Walker breakdown does not occur. There is a slowdown of the domain wall due to spin wave emission and no sudden drop in speed. For the largest values considered in [5] the domain wall speed changes from a regime of linear growth with the applied field to a regime of slower growth with increasing applied field. This last behavior is observed when both the exchange constant and uniaxial anisotropy are much smaller than hard axis anisotropy. Further numerical studies [6, 7] analyze in detail the nature of the spin waves emitted and distinguish two scenarios, depending on the relative values of the exchange and anisotropy constants. The parameter ranges studied in [5] and [7] differ, however, in both cases the Walker breakdown is not observed when the transverse anisotropy is sufficiently large. The stability of the Walker solution with respect to small perturbations has been studied recently [8] using dynamical systems techniques. The analysis of the spectrum of a perturbation to the Walker solution shows that it may become absolutely or convectively unstable before the breakdown field. This instability is found for sufficiently large transverse anisotropy and for fields larger than a critical value.

The purpose of this work is to study the dynamics of the LLG equation for a nanowire when the transverse anisotropy is large by means of an asymptotic expansion. The asymptotic expansion captures the slower relaxation dynamics of the domain wall and filters out the fast spin waves [9]. We find that the leading order asymptotic dynamics predicts a transition from a Walker type regime to a regime with slower domain wall motion. In leading order

the dynamics of the in plane magnetization obeys a reaction diffusion equation, and the perpendicular magnetization is slaved to the in-plane components. The slowdown of the domain wall in this asymptotic regime appears as a transition from a pushed to a pulled front at a critical value of the applied field.

The starting point of the calculation is the LLG equation for the magnetization. The material has magnetization  $\vec{M} = M_s \vec{m}$  where  $M_s$  is the saturation magnetization and  $\vec{m} = (m_1, m_2, m_3)$  is a unit vector along the direction of magnetization. The dynamic evolution of the magnetization is governed by the LLG equation,

$$\frac{d\vec{M}}{dt} = -\gamma_0 \vec{M} \times \vec{H}_{\text{eff}} + \alpha \frac{\vec{M}}{M_s} \times \frac{d\vec{M}}{dt} \quad (1)$$

where  $\vec{H}_{\text{eff}}$  is the effective magnetic field,  $\gamma_0 = |\gamma|\mu_0$ ,  $\gamma$  is the gyromagnetic ratio of the electron and  $\mu_0$  is the magnetic permeability of vacuum. The constant  $\alpha > 0$  is the dimensionless phenomenological Gilbert damping coefficient. We consider a thin and narrow film in the  $(x, y)$  plane, with the easy axis along its length. The strip is subject to an applied magnetic field along the easy axis  $\vec{H}_a = H_a \hat{x}$ . The film is thin and narrow so that the magnetization may be assumed [10] to depend on the easy axis coordinate,  $\vec{M}(x, y, z) = \vec{M}(x)$ . In addition, in this geometry the demagnetizing field has a local representation as an effective perpendicular anisotropy so that, as in [5–8], the effective magnetic field is given by

$$\vec{H}_{\text{eff}} = H_a \hat{x} + \frac{C_{\text{ex}}}{\mu_0 M_s^2} \frac{\partial^2 \vec{M}}{\partial x^2} + \frac{2K_u}{\mu_0 M_s^2} M_1 \hat{x} - \frac{2K_d}{\mu_0 M_s^2} M_3 \hat{z}, \quad (2)$$

where  $C_{\text{ex}}$  is the exchange constant,  $K_u$  the easy axis uniaxial anisotropy and  $K_d$  the perpendicular anisotropy.

Introducing  $M_s$  as unit of magnetic field, and introducing the dimensionless space and time variables  $\xi = x\sqrt{K_u/C_{\text{ex}}}$  and  $\tau = \mu_0|\gamma|M_s t$  we rewrite equations (1) and (2) in dimensionless form

$$\frac{d\vec{m}}{d\tau} = -\vec{m} \times \vec{h}_{\text{eff}} + \alpha \vec{m} \times \frac{d\vec{m}}{d\tau} \quad (3)$$

with

$$\vec{h}_{\text{eff}} = h_a \hat{x} + \frac{1}{2} K_{\parallel} \frac{\partial^2 \vec{m}}{\partial \xi^2} + K_{\parallel} m_1 \hat{x} - K_{\perp} m_3 \hat{z}. \quad (4)$$

where  $h_a$  is the dimensionless applied field and the dimensionless numbers that have appeared are  $K_{\parallel} = 2K_u/(\mu_0 M_s^2)$ ,  $K_{\perp} = 2K_d/(\mu_0 M_s^2)$ . Equations (3) and (4) describe the dynamics of the problem.

We are interested in the case of a perpendicular anisotropy much larger than the uniaxial in plane anisotropy. In this situation the perpendicular magnetization  $m_3$  will be smaller than the in plane components. We will also assume that the dimensionless applied field is weak. We search then for a solution of the LLG equation in the asymptotic limit  $h_a \ll K_\perp$ ,  $K_\parallel \ll K_\perp$  and therefore,  $m_1, m_2 \gg m_3$ . Let then

$$m_1 = m_{10} + \epsilon m_{11} + \dots, \quad m_2 = m_{20} + \epsilon m_{21} + \dots, \quad m_3 = \epsilon m_{30} + \epsilon^2 m_{31} + \dots,$$

where  $\epsilon$  is a small quantity. Since the perpendicular anisotropy is larger than the uniaxial anisotropy and the applied field is weak, we introduce the scaling  $K_\parallel = \epsilon \tilde{K}_\parallel$ ,  $h_a = \epsilon \tilde{h}_a$  with  $K_\perp$  of order one. The components of the effective magnetic field,  $\vec{h}_{\text{eff}} = (h_1, h_2, h_3)$  become then  $h_i = \epsilon h_{i0} + \epsilon^2 h_{i1} + \dots$  with the leading order components given by

$$h_{10} = \tilde{h}_a + \frac{1}{2} \tilde{K}_\parallel \frac{\partial^2 m_{10}}{\partial \xi^2} + K_\parallel m_{10}, \quad h_{20} = \frac{1}{2} \tilde{K}_\parallel \frac{\partial^2 m_{20}}{\partial \xi^2} \quad \text{and} \quad h_{30} = -K_\perp m_{30}. \quad (5)$$

Furthermore we introduce a slow time scale  $s = \epsilon \tau$  and notice that the leading order components of the in plane magnetization satisfy

$$m_{10}^2 + m_{20}^2 = 1 - O(\epsilon^2). \quad (6)$$

Introducing these scalings in Eq.(3) and expanding in  $\epsilon$  one obtains

$$\frac{\partial m_{10}}{\partial s} = -m_{20} h_{30} + O(\epsilon), \quad (7a)$$

$$\frac{\partial m_{20}}{\partial s} = m_{10} h_{30} + O(\epsilon), \quad (7b)$$

$$0 = -m_{10} h_{20} + m_{20} h_{10} + \alpha (m_{10} \frac{\partial m_{20}}{\partial s} - m_{20} \frac{\partial m_{10}}{\partial s}) + O(\epsilon). \quad (7c)$$

Substituting (7a) and (7b) into (7c) and using (6) we find that in leading order,

$$h_{30} = \frac{1}{\alpha} (m_{10} h_{20} - m_{20} h_{10}) \quad (8)$$

and equations (7a) and (7b) become

$$\frac{\partial m_{10}}{\partial s} = -\frac{m_{20}}{\alpha} (m_{10} h_{20} - m_{20} h_{10}) \quad (9a)$$

$$\frac{\partial m_{20}}{\partial s} = \frac{m_{10}}{\alpha} (m_{10} h_{20} - m_{20} h_{10}). \quad (9b)$$

Because of (6) we can write  $m_{10} = \cos \theta$ ,  $m_{20} = \sin \theta$ . Using (5) in equations (8-9) we obtain

$$\alpha \frac{\partial \theta}{\partial s} = \frac{1}{2} \tilde{K}_\parallel \theta_{\xi\xi} - \sin \theta (\tilde{h}_a + \tilde{K}_\parallel \cos \theta) \quad (10)$$

$$m_{30} = -\frac{1}{K_\perp} \frac{\partial \theta}{\partial s}. \quad (11)$$

Finally going back to the unscaled time variable  $\tau$  and parameters  $K_{\parallel}, K_{\perp}$ , we write the leading order magnetization components as

$$m_1 = \cos \theta, \quad m_2 = \sin \theta, \quad m_3 = -\frac{1}{K_{\perp}} \frac{\partial \theta}{\partial \tau} \quad \text{where} \quad (12a)$$

$$\alpha \frac{\partial \theta}{\partial \tau} = \frac{K_{\parallel}}{2} \theta_{\xi\xi} - \sin \theta (h_a + K_{\parallel} \cos \theta). \quad (12b)$$

Equations (12) show that the leading order dynamics is determined by the equation for the in-plane magnetization components, the perpendicular magnetization is slaved to the tangential magnetization. Equation (12b) is the well studied reaction diffusion equation, for which we know that an initial perturbation to an unstable state evolves into the monotonic front of minimal speed [12]. In order to render (12b) into the standard form we introduce the dependent variable  $u$  defined by  $\theta = \pi(1 - u)$  which satisfies

$$\alpha u_{\tau} = D u_{\xi\xi} + f(u), \quad \text{with} \quad f(u) = \frac{1}{\pi} \sin \pi u (h_a - K_{\parallel} \cos \pi u). \quad (13)$$

The diffusion constant  $D = K_{\parallel}/2$  and the reaction term  $f$  satisfies  $f(u) > 0$  in  $(0,1)$ ,  $f(0) = f(1) = 0$ . A small perturbation to the unstable state  $u = 0$  ( $\theta = \pi$ ) evolves into a traveling monotonic front of minimal speed  $c^*$  [11, 12] that joins the unstable state to the stable state  $u = 1$  ( $\theta = 0$ ). The minimal speed can be obtained from a variational principle [13] and is bounded by [12]

$$c_{\text{KPP}} \equiv \frac{2}{\alpha} \sqrt{D f'(0)} < c^* < \frac{2}{\alpha} \sqrt{D \sup f(u)/u}. \quad (14)$$

When the upper and lower bounds coincide the speed is exactly  $c_{\text{KPP}}$  and the traveling front is called a KPP or pulled front.

In the present problem Eq. (13) has the exact traveling front solution

$$u(\xi, \tau) = \frac{2}{\pi} \arctan \left[ e^{-\frac{K_{\parallel}}{D}(\xi - c_N \tau)} \right], \quad \text{where} \quad c_N = \frac{h_a}{\alpha} \sqrt{\frac{D}{K_{\parallel}}}. \quad (15)$$

This solution is not a KPP front, it is a so called pushed front. This is the front into which an initial condition will evolve it is effectively the front of minimal speed. It is not difficult to verify that as  $h_a$  increases this is not the speed of the front. For  $h_a \geq 4K_{\parallel}$  the upper and lower bounds in (14) coincide and the speed of the front must be the KPP value. The transition from a pushed to a pulled front may occur before the upper and lower bounds

coincide. In this problem for which there is an exact solution we know that the transition will occur when  $c_N = c_{KPP}$ . That is,

$$c = \begin{cases} \frac{h_a}{\alpha} \sqrt{\frac{D}{K_{\parallel}}} & \text{if } h_a \leq 2K_{\parallel} \\ \frac{2}{\alpha} \sqrt{D(h_a - K_{\parallel})} & \text{if } h_a > 2K_{\parallel} \end{cases}$$

Going back to the physical variables, we have then that the speed of the domain wall is given by

$$v = \begin{cases} \frac{1}{\alpha} \sqrt{\frac{C_{ex}}{2K_u}} \mu_0 |\gamma| H_a & \text{if } H_a < \frac{4K_u}{\mu_0 M_s} \\ \frac{2|\gamma| \sqrt{C_{ex}}}{\alpha M_s} \sqrt{\mu_0 M_s H_a - 2K_u} & \text{if } H_a > \frac{4K_u}{\mu_0 M_s} \end{cases} \quad (16)$$

In the small field regime  $H_a < H_c = 4K_u/(\mu_0 M_s)$  the magnetization profile is obtained from (15) and it is given by

$$m_1 = \tanh \left[ -\frac{(x - vt)}{\Delta} \right], \quad m_2 = \text{sech} \left[ \frac{(x - vt)}{\Delta} \right], \quad m_3 = \frac{\mu_0 M_s H_a}{\sqrt{2\alpha} K_d} \text{sech} \left[ \frac{(x - vt)}{\Delta} \right], \quad (17)$$

where the domain wall width is given by  $\Delta = (1/2)\sqrt{C_{ex}/K_u}$ . For an applied field larger than  $H_c$  we cannot construct an explicit solution for the magnetization, we can only determine the speed. The general theory of reaction diffusion equations guarantees that it is a monotonic decaying front similar in shape to (15). Equations (16) and (17) constitute our main result. These results, obtained from the LLG equation in the case  $K_{\parallel} \ll K_{\perp}$  and for a weak applied field, explain qualitatively the results of the numerical simulations [5–7] and of the stability results [8]. At low fields the speed of the front is proportional to the applied field  $H_a$  and inversely proportional to the damping coefficient  $\alpha$ . The magnetization profile and the speed share the main features of the Walker solution, the velocity shows linear dependence on the applied field and inverse proportionality on the damping constant  $\alpha$ . The Walker breakdown field  $H_W = \alpha K_{\perp}/2$  is of order one, and therefore large compared to the transition field  $H_c$ . Thus, we recover the behavior described in [5]–[8]: for sufficiently large perpendicular anisotropy the Walker solution loses stability before the breakdown field to a slower moving domain wall. When the applied field is weak and  $K_{\parallel} \ll K_{\perp}$  the numerical integrations in [5, 7] show that the speed increases slowly with the field once the Walker solution loses stability, in agreement with the results found in this work.

The asymptotic approach that we have used is based on [9], where the numerical simulations, (although for a different demagnetizing field), show that the asymptotic dynamics reproduces the relaxation dynamics of the full LLG equation, filtering out the spin waves. Reaction

diffusion dynamics has also been encountered in thin nanotubes [14], where the Walker breakdown is not observed. A transition from a fast to a slower domain wall also occurs in thin nanotubes as reported in [15, 16]. The asymptotic dynamics of the LLG equation has been studied by several authors in different limiting parameter ranges, and wave type motion governed by other evolution equations has been derived [17, 18]. In the present problem we have chosen a parameter regime for which recent numerical work has been performed and found qualitative agreement with the results reported in them.

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